Frege's Alleged Pragmatism

Abstract

Tyler Burge claims that Frege offers a pragmatic justification for believing the principles of his formalism to be both true and logical. However, I show that Frege sought no such justification because his goal was to codify the principles implicit in our inferential practice. Indeed, I argue that pragmatism cannot be reconciled with Frege's constitutive view of logic's relation to cognition. Frege thought our certain knowledge of basic logical laws secured by our status as judges. Beyond establishing the importance of properly understanding Frege's constitutive view, my interpretation shows how Frege avoids a social version of the pitfall of psychologism.

Frege's Alleged Pragmatism

> Frege's pragmatism…[plays a] role in his epistemology. [It] not only play[s] a role in accounting for understanding. [It provides] a secondary, fallible, non-demonstrative justification.
>
> Tyler Burge

According to Tyler Burge, Gottlob Frege believes that the wide applicability of his Begriffsschrift (concept-script) in mathematics and science justifies holding it true and holding it to truly codify the objective laws of logic.[1] Yet this 'pragmatic' reading of Frege's epistemology of logic distorts his enterprise. A consequence of Frege's *constitutive* conception of the relationship between logic and cognition is that logical laws neither need nor admit pragmatic (or, indeed, any) justification.

After clarifying the motivation for Burge's interpretation in section one, I argue in section two that it does not fit Frege's texts. In section three I argue further that it cannot be reconciled with Frege's constitutive view. Frege thinks our certain knowledge of basic logical laws secured by our status as judges, a position that eliminates the need to justify the Begriffsschrift while, I argue in the final section, steering clear of a social version of the pitfall of psychologism.

I

Frege views logic as the study of the relation of inference upon which scientific inquiry depends. *Prima facie,* there are two sorts of justification that one might demand from a logician who proposes a codification of the principles of demonstrative inference. First, one might want a justification that the proposed laws are *true*. Call this *the truth demand.* Second, one might want a justification that the proposed laws are *logical.* Call this *the demarcation demand.* To judge whether these demands accord with Frege's conception of his project we must consider how he introduces his formalism.

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[1] Throughout, I use 'Begriffsschrift' to refer to Frege's logical formalism and '*Begriffsschrift*' to refer to his 1879 book.

Frege calls the basic laws of the Begriffsschrift—both its axioms and its rules of inference—*selbstverständlich* or *einleuchtend* or *unmittelbar klar*, all of which are usually translated as 'self-evident.'[2] He never defines these terms. They suggest that Frege thinks his basic laws obvious. To understand an axiom is to be wholly convinced of its truth. But obviousness and conviction are matters of psychology. Frege cannot mean to invoke obviousness as a justification in response to the truth demand, for then—like the psychological logicians he steadfastly opposes—he would be allowing psychology to cloud his logical investigation.[3] One judge may doubt what another finds obvious. One's confidence in an alleged truth may be eroded. Nor does attributing obviousness to his codification answer the demarcation demand. Had Frege concluded that any general law he found obvious was logical, then his account of properly logical laws would have relied upon his own psychology.[4] Some interpretive work is demanded if we are to avoid condemning Frege himself with psychologism.[5]

Burge (1992, 1998) absolves Frege by crediting him with a nuanced understanding of self-evidence. He claims that Frege distinguished the psychological and subjective impression of obviousness from the non-psychological and objective property of self-evidence enjoyed by the true laws of logic. In Burge's view, Frege would accept that principles he held to be self-evidently true might turn out to be false, and that principles he held to be primitive logical truths might turn out to be true but in need of proof on the basis of still more primitive (and actually logical) truths.

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[2] Frege, *The Foundations of Arithmetic* (FA), §5, §90; *Posthumous Writings* (PW), 39, 208-210.
[3] Frege is explicit that psychological facts cannot justify logical truths (*Grundgesetze der Arithmetik Vol. 1* [GGZ1], xvii). As Wittgenstein later put it, 'if the truth of a proposition does not *follow* from the fact that it is self-evident to us, then its self-evidence in no way justifies our belief in its truth' (*Tractatus Logico-Philosophicus*, §5.1363).
[4] Wittgenstein famously observes: 'it is remarkable that a thinker as rigorous as Frege appealed to the degree of self-evidence as the criterion of a logical proposition' (*Tractatus*, §6.1271). But this objection is not decisive, because, as I argue in section two, the demarcation demand is irrelevant to Frege's actual project.
[5] Frege's claim that 'we cannot accept a thought as an axiom if we are in doubt about its truth' (PW, 205) might suggest that he thought *rationally indubitable* obvious truths self-evident. But this modification does not provide a non-psychological way of meeting either demand, since truths may *appear* rationally indubitable without being so.

Recent work by Robin Jeshion (2001) clarifies how such an interpretation shapes the truth and demarcation demands. Jeshion admits that Frege never gives the various terms univocally translated as 'self-evident' different technical meanings, and that no putative explication fits all of his texts precisely (953). (Burge also confesses that he has not 'found consistent differences in Frege's usage' of these terms [1998, 335.]) Nevertheless, Jeshion offers the following explication of Frege's view:

> A true proposition *p* is *selbstverständlich* if and only if *p* is foundationally secure, yet *p*'s truth is not grounded on any other truth. (949)[6]
>
> A true proposition *p* is *self-evident* [*einleuchtend*, *unmittelbar klar*] if and only if clearly grasping *p* is sufficient and compelling basis for recognition of *p*'s truth. (953)

Jeshion characterizes Frege's logicist project as the 'search for foundational arithmetical axioms [that are] both *selbstverständlich* and self-evident,' and believes that, since all self-evident propositions are *selbstverständlich,* Frege 'relied on judging propositions to be self-evident as part of his method for identifying a foundation for arithmetic' (939).[7] But one must, in turn, rely upon subjective obviousness as a guide to judging propositions self-evident. Obviousness is a fallible guide, because not all propositions one finds obvious are self-evident, and one may not find all self-evident propositions obvious (if, for instance, one lacks the requisite understanding of component concepts). Nevertheless, Jeshion thinks obviousness is 'normally necessary' for identifying primitive truths (967).[8]

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[6] Jeshion writes that 'a *ground* of a true proposition indicates from whence the proposition's truth derives' (945). Thus, the truth of a *selbstverständlich* thought does not derive from any other thought.

[7] Frege, according to Jeshion, does not believe the converse implication. An example is the equation Frege gives in FA, §5: 135664+37863=173527. All true arithmetical equations are disguised definitions, and so, are *selbstverständlich*. They lack the generality required to stand as axioms, but are nevertheless foundationally secure (950). To Jeshion's Frege, 'if one grasps a self-evident proposition *p* yet fails to thereby recognize its truth, the failure to do so implies a deficiency in conceptual understanding or rationality' (956). Since a fully rational agent can understand this equation and yet (prior to checking it) refrain from endorsing or rejecting it, it is not self-evident.

[8] Only 'normally,' because pragmatic considerations may sometimes suffice. Frege's Basic Law V may be an example; see note 21 below.

Under Jeshion's explication, Frege's attribution of 'self-evidence' to his basic laws is to be understood normatively, not psychologically. Logical laws are self-evident to, as Burge puts it, an 'ideally rational mind' (1998, 340), not Frege's own. Unlike Wittgenstein, whom Jeshion characterizes as 'preoccupied' with certainty, Frege 'never took particular proposed foundations as certain' (2001, 972-3). Nevertheless, Frege was confident that he had successfully codified logic (GGZ1, xxvi). If Burge and Jeshion are correct that he thought his method for identifying foundational principles fallible, then the truth and demarcation demands become pressing. Frege owes readers who fail to find the Begriffsschrift obvious a justification for his conviction in its truth and logicality.

Burge claims Frege transmits his confidence in two ways. The first is via introductory 'semantic claims' regarding each individual rule and axiom (1998, 316). Burge acknowledges that Frege could not have thought his axioms, as *selbstverständlich* truths, provable on the basis of more fundamental considerations. So the semantic claims cannot be intended to justify the truth of the axioms. Rather, they serve to expose their self-evidence to a readership unfamiliar with Frege's novel notation. In contrast, in §14 of *Grundgesetze*, Burge interprets Frege as developing a genuine semantic argument to justify the soundness of the Begriffsschrift rule *modus ponens* (320). By reflecting on the meanings of the material conditional, the horizontal, and the judgment stroke, one should recognize that the transition licensed by the notation's inference rule from true sentences whose syntactic forms are a conditional and its antecedent to the sentence whose syntactic form is the conditional's consequent is truth-preserving.[9]

[9] Burge acknowledges that this soundness argument is unsystematic by modern lights, and finds Frege's pragmatic considerations his more significant contribution to epistemology of logic (1998, 344). But there is reason to think that the truth predicate necessary to develop this soundness argument is not part of Frege's conception of logic. For criticisms of Burge's interpretation of Frege's 'semantic claims,' see Weiner (1990, 170-175; 1995, 364; 2008, 427-50) and Ricketts (1996, 121-40).

Yet although such bottom-up semantic considerations might transmit confidence in the truth of individual laws, they do little to transmit confidence that the formalism adequately captures the principles of demonstrative inference. We now learn predicate logic *as* logic in introductory courses, but Frege's readers would have found many of his innovations (such as using the artificial material conditional rather than the syllogistic forms to express universal categoricals) obscure. To meet this problem, Burge reads Frege as offering a second, pragmatic, top-down justification for the Begriffsschrift in its entirety:

> What is original about [Frege's] position is not his view that a thought might be self-evident but not seem self-evident—self-evident but not obvious to an individual. It is not his idea that subjective obviousness or subjective unobviousness might submit to reversal through deeper conceptual development and understanding. What is original is his integration of these traditional views with his deep conception of what goes into adequate understanding. This conception rests on his method of finding logical structure through studying patterns of inference…[U]nderstanding logical structure derives from seeing what structures are most fruitful in accounting for the patterns of inference that we reflectively engage in…Frege's pragmatism…[plays a] role in his epistemology. [It] not only play[s] a role in accounting for understanding. [It provides] a secondary, fallible, non-demonstrative justification. (1998, 344)

What sort of 'justification' is Burge thinking of? Reflecting upon our inferential practice cannot justify that the axioms of the Begriffsschrift are true, because (to repeat) those axioms are meant to express objective logical laws that are *selbstverständlich*. Rather, Burge writes, such reflection provides 'justification for believing' the formalism to truly codify logic, its axioms to be true, and its rules truth-preserving (1998, 324; cf. 1992, 648).[10]

Burge's idea is that, since mathematics and the more rigorous sciences are saturated with the quantificational inferences the Begriffsschrift makes explicit, there is pragmatic reason for Frege's readers to think that the notation brings out deep, structural relationships among

[10] This is already a sign that Burge's interpretation has gone wrong. Frege's insistence that logic be divorced from psychology makes attributing any notion of 'justification for believing' to him puzzling. What an individual believes is a matter of her psychology, but what is justified, according to Frege, are mind-independent thoughts.

objective thoughts that subsist in a 'third realm.'[11] Capturing this structure—which is *logical* structure, structure in virtue of which the truth of some thoughts guarantees the truth of others—is the target of a codification of the principles of demonstrative inference. So as readers come to appreciate the Begriffsschrift's power to formalize reasoning that they informally judge valid, their confidence in the truth and logicality of the formalism ought to increase. This justification is 'fallible,' however, because our scientific theories—and the patterns of inference we think legitimate—evolve with our discoveries. In the course of ongoing inquiry we may discover thoughts whose contents or structural relationships cause us to adopt (or abandon) an inferential pattern that the Begriffsschrift omits (or enshrines).[12]

In Burge's view, the Begriffsshcrift's secondary, pragmatic justification also paves the way for inquirers to recognize its primary justification: the objective self-evidence of its basic laws. Readers will improve their grasp of the Begriffsschrift's novel concepts in the course of becoming familiar with its various applications, eventually coming to find axioms that they may initially have found obscure to be subjectively obvious. In turn, this subjective obviousness gives them reason to believe that Begriffsschrift axioms express self-evident truths, or, as Jeshion puts it, that the Begriffsschrift 'mirrors the natural ordering of truths' (2001, 961).[13]

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[11] Frege writes that thoughts 'belong' to a 'third realm' in 'Thought' (T), 337 [69] (I shall use brackets throughout to indicate the original pagination). Burge argues that Frege views Platonist ontology as a substantive theory necessary to explain the objectivity of communication (1992, 641-645).

[12] That a Begriffsschrift law might be abandoned does not contradict Frege's insistence that the laws of logic 'are boundary stones set in an eternal foundation, which our thought can overflow but never displace' (GGZ1, xvi). On Burge's interpretation, the Begriffsschrift is a fallible attempt to express the immutable logical laws. It might turn out to be incomplete if Frege missed a valid inferential pattern. And its axioms might be false if he made a mistake.

[13] Jeshion's mirror metaphor has the effect of emphasizing Frege's Platonist remarks. For it suggests he thought a codification of logic 'reflects' (and is thus secondary to) relationships among thoughts in the third realm. But since Frege thought a logical truth could be legitimately expressed as an axiom in one codification and a theorem in another, he would accept that two codifications may not 'reflect' each other (*Begriffsschrift* [BGS] 29; PW, 206). This casts doubt upon the appropriateness of Jeshion's metaphor. If third-realm relationships cannot be uniquely mirrored, what grounds could we have for thinking that they subsist independently to be mirrored at all? The divisive issue of how to interpret Frege's Platonism (see Ricketts [1986, 65-95] and Weiner [1990, 216-218]) accordingly extends to epistemology. For without ontological priority—or even if Frege viewed logic and ontology

II

Burge's interpretation, though charitable, is mistaken. It creates a gap between our reflective exercise of the capacity for inferential judgment and our knowledge of the structural, logical relationships that exist among third-realm thoughts. According to Burge, Frege fallibly roots the latter in the former, on the pragmatic basis that 'pure mathematical practice *works*' (1998, 330, original emphasis). But the existence of this gap makes intelligible a question that Frege would have found absurd: might the principles implicit in our inferential practice *fail* to be grounded in the structure of third-realm thoughts, the objective 'laws of truth' (PW, 148)? The question is absurd because as judges we display our certain grasp of objective truth (the constitutive goal of judgment) in our capacity for demonstrative inference.

In my view, capturing the principles underlying our inferential practice *is* Frege's goal, not a defeasible reason for thinking that we have obtained a *further* goal. The Fregean logician aims to elaborate the foundation of our evident capacity for judgment, and thus has no need to posit Burge's justificatory bridge between contingent human inquiry and eternal relationships among third-realm thoughts. Neither the truth nor the demarcation demands pertain to Frege's project. Instead, readers who understand Frege's notation are already positioned, as judges, to evaluate whether it achieves its aim of making explicit the inferential principles upon which the practice of judgment relies.[14]

Frege turns to scientific inquiry to uncover the desiderata for his concept-script, which he intends to be the universal language of justification (BGS, vi). It should be possible to construct an unambiguous representation of each candidate for judgment in the Begriffsschrift, so as to

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as 'mutually entangled' (Burge 1992, 644 n16)—third-realm relationships do not constitute an independent standard against which our codification can (and must) be evaluated. I return to this point below.

[14] My reading belongs to a family originating with van Heijenoort (1967), who first emphasized important differences between Frege's conception of logic and our own.

minimize communication failures between judges. Further, the Begriffsschrift ought to make explicit the inferentially relevant content of each thought, so that in cases of disagreement the considerations that would settle the dispute are transparent. Having accomplished these goals to his satisfaction, he proposes that scientists make use of the Begriffsschrift when they need to better understand a disputed proof (BGS, v-vi), a feature of his thought that gained prominence after the discovery of the contradiction shattered his faith in logicism.[15]

Formalizing a judgment within the Begriffsschrift ought to make its justification plain. But the truth of the basic principles of Begriffsschrift cannot be justified by appeal to other truths. Any attempt to do so would rely on the truth of the very principles of inference it sought to justify, and would hence be viciously circular. This renders the truth demand illegitimate. As Frege conceives of his project, there is neither a need to provide nor a possibility of providing judges with a *justification* for why they should hold the Begriffsschrift true. When teaching it (given its novelty), there is only a need to draw out how judges are already positioned to find its principles true and basic (i.e. self-evident), and to convey its usefulness for their inquiries.

Moreover, Frege neither gives nor needs to give a response to the demarcation demand. He lacks a substantive account of what makes a truth logical. This may seem surprising. Such an account is surely required, one might object, for Frege's logicist project, and to act as a standard against which to evaluate his formalism.

These objections both mistake Frege's project. Frege creates the Begriffsschrift to help him to establish logicism, the claim that a gap-free proof for any arithmetical theorem can be constructed using only logical laws and definitions. So he does not need a general criterion by

[15] In 'What may I regard as the result of my work?' Frege writes, 'It is almost all bound up with the concept-script' (PW, 184).

which to determine whether a given truth is a logical truth. He needs only show that a gap-free proof for any arithmetical theorem can be derived from the Begriffsschrift's basic laws.

Secondly, the demarcation demand presupposes an antecedent grasp of the properly logical domain to which one holds up a proposed codification. Yet Frege's texts give no indication that he thought himself, or his readers, so equipped. Richard Heck has plausibly argued, for example, that Frege knew that there were undecidable sentences (2010, 348), so he could not have thought that all truths expressible using only the topic-universal vocabulary of the Begriffsschrift (which is one account of what makes a truth 'logical') were derivable within the Begriffsschrift.[16] Absent a criterion of 'the logical,' a real definition of its essence, the demarcation demand is not cogent. But crucially, it does not follow that the Begriffsschrift cannot be evaluated as 'logical,' for Frege defends the Begriffsschrift as meeting a *nominal*, not a real, definition of logic. He describes his concept-script as a 'fresh approach to the Leibnizian idea of a *calculus ratiocinatur'* (PW, 10), and defends it as more worthy of the *name* logic than its competitors.[17] In addition to pointing out its pragmatic advantages of flexibility, generality, and simplicity (GGZ1, xi-x; PW, 35), he formally proves core arithmetical theorems to showcase its expressive power (PW, 38). (These pragmatic considerations do not, as Burge claims, play a *justificatory* role for Frege, but a *pedagogical* one. They motivate a careful examination of the Begriffsschrift.) Frege believes that Begriffsschrift proofs of the basic truths of arithmetic suffice to establish logicism, and is moderately confident that, with suitable expansions of its vocabulary, his formalism can be employed to express the proof of any scientific claim (BGS, vi). But, he also acknowledges that other rules of inference might have to be introduced into the

[16] For example, it is either true that all objects are value-ranges, or that there is an object that is not a value-range; but neither claim is derivable within the Begriffsschrift. See also (GGZ1, §10), where Frege states that the claim that The True is the extension of the concept under which only The True falls is not derivable, but nevertheless stipulates that it is true.

[17] Frege argues his formalism is superior to both Peano's (*Collected Papers* [CP] 234-248) and Boole's (PW, 9-52).

language in order to give formal gap-free proofs for some scientific truths (PW, 363). This shows that Frege did not intend his Begriffsschrift to be the final word on logical truth, and that he did not intend readers to evaluate it by judging whether it captured *every* truth they antecedently knew to be logical. Rather, Frege dubs the Begriffsschrift 'logic' as an honorific: perceptive readers will appreciate that he has created a language that (he believes) meets the desiderata for an ideal language for science, the truths derivable from which thus deserve to be called 'logical.'

The textual evidence Burge provides for his alternative justificatory reading of the epistemic significance Frege assigns to reflecting on our inferential practice is not credible. He cites Frege's claim that 'definitions show their worth by proving fruitful' (FA, §70; cf. PW, 33) as evidence that 'Frege appeals to mathematical practice...as a way of confirming the worth, and seemingly the correctness, of the definition' (1998, 331). But there is no evidence that Frege thought that the truth of a definition was 'confirmed' by the reader's evaluation of its fruitfulness as Burge's addition of 'correctness' to Frege's text implies. For example, Frege argues that his definition of Number in terms of extensions is fruitful not only because it allows for the formal proof of arithmetical truths, but because extensions can be repeatedly and gainfully employed in subsequent deductions involving complex arithmetical structures.[18] This applicability confirms the worth of the definition but neither gives, nor is it intended to give, justificatory support for its truth.

Burge claims that Frege's recommendation that readers familiarize themselves with the Begriffsschrift shows that he believed readers' 'acceptance of [the truth of his basic principles might depend on] detailed mastery of the system' (333). Further, Burge asserts that Frege

[18] '[Every element in the] really fruitful definitions in mathematics' is, Frege writes, 'intimately, I might almost say organically, connected with the others,' allowing for inferences that 'cannot be inspected in advance' (FA, §88). His example is the epsilon-delta definition of continuity. When one proceeds to prove topological theorems, one will continually return to this definition to argue in terms of a function's limits. Similarly, once Frege proceeds to prove arithmetical theorems, he repeatedly returns to his definition of Number to argue in terms of extensions.

believed that the laws of his Begriffsschrift might be flawed. Neither claim is supported by the passages Burge quotes:

> If anyone should find anything defective, he must be able to state precisely where, according to him, the error lies: in the Basic Laws, in the Definitions, in the Rules, or in the application of the Rules at a definite point. (GGZ1, vii)

> I myself can estimate to some extent the resistance with which my innovations will be met, because I had first to overcome something similar in myself in order to make them...After one has reached the end [of the book], he may reread the Exposition of the *Begriffsschrift* as a connected whole...[T]he suspicion that may at first be aroused by my innovations will gradually be dispelled. The reader will recognize that my *basic principles* [Burge's emphasis] at no point lead to consequences that he is not himself forced to acknowledge as correct. (xi-xii)

Burge claims that in the first passage, 'the language clearly suggests that [the] defect might in principle lie not only in the ordering or in the proposed axioms not being logical, but even in proposed basic principles not being true or sound' (333). He claims that the second passage shows that Frege believed a critic might initially *doubt* that a particular axiom was true (or a rule truth-preserving), but become convinced of its truth by working through the book (334). But this interpretation neglects the context of these passages. Frege had been greatly frustrated by how poorly *Begriffsschrift* had been understood by those of his contemporaries who read it, including its few reviewers.[19] Just prior to the second passage Burge quotes, Frege complains that both *Begriffsschrift* and *Foundations* have been unfairly ignored (GGZ1, xi). Much of the introduction to *Grundgesetze* is thus designed to ward off shallow, confused criticism.

Restoring this context exposes that Frege's aim here is to dissuade readers from prematurely discounting his innovations. The language of the first passage does not 'clearly suggest' that Frege believed his basic principles might be false, but only that he anticipated *others'* (erroneous) objections to his system. Frege's hope is that in the course of making their

[19] See Dummett (1973, xxxi) and Currie (1986, 79).

putative objections precise, his critics will realize that they are mistaken.[20] Similarly, the 'suspicion' Frege imagines in the second passage is not doubt about the *truth* of his basic principles, but rather doubt about the *usefulness* of his peculiar 'innovations.' This doubt is what Frege thinks will be 'gradually dispelled' as readers reflect upon his worked examples, realize that the Begriffsschrift's novel concepts are actually precise explications of the vague concepts they already use, and appreciate that deriving proofs within the formalism clarifies the inferential patterns upon which they tacitly rely.

Burge also highlights Frege's prescient doubt about Basic Law V:

> A dispute can break out here, so far as I can see, only with regard to my fundamental law concerning value-ranges (V), which has not yet perhaps been expressly formulated by logicians, although one has it in mind, for example, when speaking of extensions of concepts. I hold it to be purely logical. (GGZ1, vii)

Burge takes this remark to show that Frege was 'uneasy about [Basic Law V's] truth' (1998, 337, n21). Since Frege advanced the law as an axiom, this suggests to Burge that Frege must have thought it a self-evident truth that he himself did not (yet) find subjectively obvious. Nonetheless, he took himself pragmatically justified in holding it true because of its place in the Begriffsschrift, and the advantages of the Begriffsschrift as a whole.[21]

Even granting Burge this much, it would not follow that Frege took a similarly fallible attitude toward his other basic laws and inference rules. Indeed, that this is the sole dispute Frege can conceive of instead suggests that the rest of his formalism is certain.[22] But nor is Burge's the

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[20] Later, Frege wryly challenges any dissatisfied critic who cannot articulate a specific problem with the Begriffsschrift to 'just try' to construct a better codification, rather than allowing vague uneasiness to prejudice them against its value (GGZ1, xxvi).

[21] This reading dovetails with Frege's reaction to Russell's discovery of the contradiction: 'I have never disguised from myself its lack of the self-evidence that belongs to the other axioms and that must properly be demanded of a logical law…I should gladly have dispensed with this foundation if I had known of any substitute for it. And even now I do not see how arithmetic can be scientifically established…unless we are permitted—at least conditionally—to pass from a concept to its extension' (*Grundgesetze der Arithmetik*, *Vol. 2* (GGZ2), 253). On Burge's interpretation, Frege elected to *hold* Basic Law V true despite its dubiousness because of his logicist conviction.

[22] It is worth emphasizing that Frege presents Russell's contradiction by using Basic Laws I-IV and Begriffsschrift inference rules to prove that no second-level function can satisfy the condition laid down in Basic Law V (GGZ2,

only way to understand Frege's imagined dispute. It is equally charitable to interpret Frege as uneasy not about Basic Law V's *truth*, but about its role in clarifying the principles of inference. The value of other novel features of his formalism (e.g. taking truth-values to be objects) would swiftly become apparent to anyone who familiarized herself with the notation, and who reflected on the simplicity and elegance of the resulting proofs. But Basic Law V functioned to streamline the proofs of mathematical theorems that were being colloquially deduced using a variety of inference patterns.[23] Frege might have anticipated that an inquirer who doubted or was unfamiliar with some of those patterns could fail to grasp the value of Basic Law V. Moreover, he allowed that another logician might discover a simpler way than he to formalize the diverse inferences.[24] These reasons may explain why Frege assessed Basic Law V differently to his other basic laws.

Burge wanted to exonerate Frege from the charge of having given a psychological answer to the truth and demarcation demands in appealing to self-evidence. But describing basic laws as self-evident is objectionable only if one thereby intends to be answering the truth and/or demarcation demands. In my view, Frege answers neither demand, because neither demand is legitimate on his conception of logic. I have argued that this reading is equally charitable, and better comports with Frege's texts.

256-260). He expresses none of the doubt about his other basic laws that might be expected if Burge were correct, and Frege thought that his grasp of those laws was as fallible—and now as demonstrably in jeopardy—as his grasp of Basic Law V. Rather, his attitude is consistent with thinking that although he has certainly succeeded in codifying *some* of the principles of inference in the rest of Begriffsschrift, he has made an isolated mistake.

[23] For more on the mathematical inferences licensed by Frege's contemporaries, see Wilson (1995).

[24] Indeed, Frege's contemporaries had not reached a consensus about how to understand extensions (as Burge notes, 'the dispute between Frege and the iterative set theorists suggests that there was a fundamental doubt about the viability of the notions (respectively) of set and of extension' [1998, 337, n21]). Perhaps Frege worried that, if mathematicians decided to impose restrictions upon extensions, his definition of Number in terms of them (for which Basic Law V allowed) would no longer be fruitful. Or again, Ricketts (1997, 200, 210-1) argues that Frege only had a clear grasp of the existential generalization of Basic Law V, rather than a particular second-level function that could serve as the courses-of-values function. A constructive mathematician might object to introducing a function in this way.

III

Although both the truth and demarcation demands are foreign to Frege, he does engage a closely related question regarding the nature of logic: how does logic normatively govern all rational inquiry? Frege's answer is that to count as reasoning about a domain one must accept the basic laws that govern it as binding, and that basic logical laws are maximally general, articulable using only topic-universal notions necessary for the expression of knowledge in any special science.[25] Since basic logical laws accordingly govern every act of judgment, not merely those in a particular branch of science, Frege believes they deserve the name 'laws of thought' and express *constitutive* norms of inquiry, '[prescribing] universally how one should think if one is to think at all' (GGZ1, xv).

Among scientific laws, Frege thus awards basic logical laws a privileged status with respect to cognition. We may engage in further inquiry with a quack who insistently makes wild inferences about some special science, and who we judge to wrongly reject one or more of its basic laws, because, so long as we avoid that science and its subsidiaries, her valid inferences make her recognizable as a member of our inferential community. But things are different with logic, because logic counts *every* domain of inquiry a subsidiary. There is no avoiding the scope of its maximally general laws.

Frege makes this point vivid in a passage where he imagines encountering 'humans or other beings' whose 'laws of thought directly contradict our own [so that they are] frequently led to contrary results [to us] in practice.' He judges these beings not merely wrong, but rather to exhibit 'a hitherto unknown kind of madness' (GGZ1, xvi). Their resolute anti-logical behavior undermines our ability to consider them fellow rational judges. Unlike psychological logicians who view logical laws as empirically discoverable generalizations of patterns of thought, and

[25] Frege writes, 'logic is the science of the most general laws of truth' (PW, 128). See also (FA §14).

who upon meeting such beings would thus conclude that different 'logical' laws are valid for them, Frege insists that genuine laws of thought are laws of *truth*, '[prescribing] how judgments should be made, wherever, whenever, and by whomever they may be made' (GGZ1, xvii). This exposes Frege's deep commitment to *our* practice of inquiry being *the* practice of inquiry. The act of judgment—which he elucidates as 'the acknowledgment of the truth of a thought' (T, 329 [62])—requires grasp of objective truth. One counts as grasping objective truth only if one is subject to the basic laws of truth in one's inferential practice, that is, if one is capable of recognizing how some truths depend upon others, of stringing judgments together in truth-preserving inferential chains. A constitutive part of this capacity for valid demonstrative inference is being disposed to judge basic logical laws true.

Frege thus adopts a constitutive account of logic's relationship to cognition:

CON: The ability to reason demonstratively is a constitutive, necessary condition for being a judge.[26]

Burge cannot, I believe, adequately account for CON. To cash out 'the ability to reason demonstratively' requires referencing a set of norms to which those possessing the ability are properly sensitive. Burge's Frege accepts that Begriffsschrift laws may be rationally deniable—not only because they may be unobvious to the novice, but because they may not even be true. It cannot be, then, that endorsing the norms grounded in *this* codification is a constitutively necessary criterion for being a judge. Burge is thus forced to read Frege's constitutive view as a claim about judges' sensitivity to objective, third-realm logical laws. These third-realm laws are the ones that fully rational judges cannot sincerely deny, and which all judges must display some grasp of in their inferential practice.

[26] Although I lack space to explore this point here, there are passages that suggest Frege's constitutive view extends even further, such as his claim that upon denying a logico-arithmetical law 'even to think at all seems no longer possible' (FA §14). Perhaps systematic anti-logical behavior that erodes our confidence that others are fellow judges should also, upon reflection, erode our confidence that they are capable of what we call thinking and asserting.

In fact, Burge thinks Frege's constitutive view enters his epistemology by providing another sort of justification. In addition to our pragmatic justification for holding Begriffsschrift laws to express true, third-realm logical laws, we now have a justification 'entitling' us to hold the third-realm logical laws themselves true:

> [Frege holds that] acknowledgment of certain laws of truth is necessary for having reason and for engaging in non-degenerate thinking and judging. One is rationally entitled to judge the primitive laws of logic to be true because the nature of reason—and even non-degenerate judgment—is partly constituted by the prescription that one acknowledge at least the simple and basic laws of truth. To put it crudely, reason and judgment—indeed mind—are partly defined in terms of acknowledging the basic laws of truth. (1992, 648-649)

Burge goes on to write that 'the laws of truth are independent of judging subjects' and that they are not 'constitutively dependent on…practice' (649). How, then, do we know that we *are* displaying some grasp of those laws in our practice?

Once again, Burge offers pragmatism as a response. According to Burge, Frege believes reflecting upon our inferential practice not only uncovers objective laws of logic, but that it pragmatically justifies us to hold that the laws we thus uncover *are* objective truths. Yet on this view, as I noted above, we can never be sure that we have bridged the gap between colloquial inferential practice and the third-realm truth-guaranteeing relationships that constitute the framework of rational judgment. Suppose we all agree that a given codification captures the norms of our inferential practice. Suppose we also agree that our practice works (it seems to allow us to establish truths, negotiate the world, etc.). Nevertheless, in Burge's view, our pragmatic justification for believing the codification to express the third-realm laws of logic is necessarily fallible. Our inferential practice could be systematically mistaken.

One might take advantage of the space opened up by Burge's gap to pose skeptical threats. For instance, since Burge thinks that practice-independent laws of logic bear the constitutive relationship to cognition, the fact that we cannot be certain that we are displaying a

grasp of these laws in our practice robs us of certainty that we are judges. Our subjective impressions to that effect are no guarantee that we grasp objective third-realm laws. However great our confidence in the truth of laws particularly basic to our inferential practice—such as the law that every object is self-identical—our confidence is only based in our fallible judgment about their apparent obviousness and our practice's apparent efficacy. To Burge's Frege, the belief that our inquiries are subject to the genuine laws of truth remains forever tentative.

Frege's unflinching attitude toward the anti-logical beings that he imagines fails to cohere with this interpretation. We would be prevented, Frege says, from supposing beings right to reject a basic principle of our inferential practice, and also prevented from doubting whether we or they are right (GGZ1, xxvii). We would be entitled, that is, to judge such beings wrong. On Burge's interpretation, this amounts to an entitlement to hold the practice of such beings wrong on the grounds that we find our laws subjectively obvious and pragmatically validated. But it nevertheless remains an open possibility that the anti-logical beings are right about the third-realm objective laws of logic. Since the correctness of our codification of logic is fallibly certified (but not guaranteed) by our pragmatic evaluation, we cannot close off this skeptical possibility. We are left begging the question: we are rational judges while beings that seem anti-logical to us are mad (even though we presumably seem anti-logical to them).

Burge correctly notes, of course, that Frege was unconcerned by skepticism, valorizing his attitude as part of the 'advanced spirit of the age' (1998, 329). But it becomes puzzling on Burge's interpretation why Frege was *not* concerned by skepticism, given that a skeptical threat is so strongly invited by the way Burge must read Frege's discussion of anti-logical beings. Even supposing this skeptical threat could somehow be answered, however—perhaps by insisting that we lack positive grounds to doubt the laws of our codification from the mere conceivability of

anti-logical beings who deny them—Burge's gap between our reflective ability to reason demonstratively and the structural relationships that obtain between third-realm thoughts remains.

The root of Burge's mistake is the thought that we have a grasp of truth that is independent of our inferential practice. If we hold separate objective truth as, on the one hand, the constitutive goal of rational judgment, and, on the other hand, that at which we aim in our inferential practice, it becomes intelligible to question whether the guiding principles of our practice could be false. In my view, Frege thinks our sole understanding of truth comes *through* our inferential practice, as the name of our epistemic goal. Our ability to reason demonstratively is thus inseparable from our grasp of truth. The Fregean logician—who, as Burge rightly notes, develops her codification by considering the patterns of inference in which we reflectively engage—is concerned to explicitly identify and codify the norms governing our practice, not to quest for practice-independent objective laws of truth of whose grasp she can never be certain. Her project is closer to home.[27]

My interpretation of Frege's project better accords with his constitutive view. The 'ability to reason demonstratively' in CON is what *we* do when engaging in scientific inquiry, making CON the claim that judges must be properly sensitive to the basic principles governing our practice. There is no possible question about whether principles of our inferential practice are correctly grounded in an inaccessible third-realm standard. Rather, our reflective equilibrium about the basic principles that govern our inferential practice functions as the shared, objective standard for the correctness of judgments. To be ungoverned by these principles is to be

[27] We do well to recall Frege's admonition: 'what logic is really concerned with is not contained in the word "true" at all but in the assertoric force with which a sentence is uttered' (PW, 252). So the Fregean logician aims to surface the general principles implicit in the capacity for demonstrative inference, the capacity in virtue of which our sincere utterances carry the weight of assertion. Frege himself sought additionally to show arithmetic derivable solely from those principles.

incapable of fully rational judgment, a fact that justifies dismissing beings whose practice contradicts our own.

Frege's constitutive view is thus far more plausible than it may seem from other recent accounts. William Taschek, for example, writes that Frege believes all judges must 'acknowledge' the basic laws of logic, but declines to 'explicate with any precision what exactly this acknowledgement consists in,' on the grounds that doing so would be 'very tricky' (2008, 384). Taschek proceeds to note that acknowledgment must 'involve one's possessing a capacity to recognize—when being sincere and reflective, and possibly with appropriate prompting—logical mistakes in one's own judgmental and inferential practice and that of others' (384). If we conceive of this 'appropriate prompting' as a flat-footed insistence that judges must simply *see* logical mistakes once they are pointed out because logical laws are self-evident, Frege's view looks both crude and too demanding. On the contrary, Frege's constitutive view is subtle enough to make various mistaken reasoners intelligible as fellow judges who are attempting—just as we are—to recognize true thoughts, each of whom may err in different ways and for different reasons.[28] Different mistakes will warrant different programs of education or rehabilitation.

For instance, just before calling those whose inferential practice systematically contradicts our own mad, Frege charitably supposes that beings who occasionally make judgments that contradict one of our logical laws might 'not as we do immediately recognize certain truths, but have to rely perhaps on the longer path of induction' (GGZ1, xvi). His idea is that such psychologically limited judges will only endorse a logical law as a general truth after explicitly considering a sufficient number of cases for them to consider it inductively established.

[28] 'What is true,' Frege writes, 'is true independently of our recognizing it as such. We can make mistakes' (PW, 2). Objecting to the fallible, pragmatic justification Burge attributes to Frege is not to implausibly attribute infallibility about judgment to Frege. Judges may intelligibly make various mistakes—even basic logical mistakes—but nevertheless to *count* as judges must engage in an inferential practice that is governed by logical laws, which they can thus be brought to realize they are already committed to accepting as true without needing a 'justification'.

If they have yet to carry out this empirical work, they may reasonably (though, as we know, wrongly) judge contrary to a logical law. In so doing they will doubtless soon get into difficulties. Given time, we can instruct such beings by indicating their ample evidence for the logical law, and by pointing out that violating it has consistently gotten them into trouble. We are entitled to judge these reasoners cautious, or perhaps simple, but not crazy. Similarly, Frege would have no difficulty making sense of bold reasoners from some of whose inferential patterns we cautiously refrain, locally eccentric and locally foolish reasoners who endorse invalid inferences in esoteric or difficult fields respectively (while making few such errors elsewhere), and formally inept reasoners who fail introductory logic classes.[29] All of these judges, *simply as judges*, can be brought not only to endorse particular formulations of basic logical laws but also to recognize that basic logical laws express norms of the very practice in which they, and we, are engaged. Frege's constitutive view only denies the capacity for rational judgment to those beings who systematically and unrepentantly violate our inferential patterns, and who thereby prevent us from thinking them members of our inferential practice.[30]

Understanding Frege's constitutive view is necessary to understand his conception of logic, but its interest extends beyond historical scholarship. In my view, Frege's insight that it is what *we* call logic that *is* logic, that *our* reflectively valid inferences *are* valid inferences, cements our epistemic foundations and earns him an important place in understanding logic's relationship to knowledge.

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[29] Had Frege been aware of subsequent formal developments, he could likewise have made sense of those competing logicians (e.g., intuitionists subscribing to Heyting's calculus) whose alternative (but not contradictory) inferential practices could be coherently explained as issuing from a guiding principle (such as constructivism). Another example is Niko Kolodny and John MacFarlane's (2010) argument for a restriction on using *modus ponens* in cases of conditional obligation.

[30] Of course, we may vacillate in our judgment as to whether, or how, the quirky judges we have just met are deformed, and nothing about Frege's constitutive view rules out the possibility of devious beings who try to mislead us by pretending to reject a basic logical law. Nevertheless, we have a defeasible reason for doubting that a being who resolutely violates the inferential pattern codified by one of our basic logical laws is a judge.

IV

However, Peter Sullivan has objected that any conception of logic making ineliminable reference to our inferential practice cannot be Frege's:

> Grasp of the kind of norm [that] truth is excludes any…essential reference to 'linguistic practice', as much as, and in the same way as, it excludes reference to human psychological processes. [Footnote:] That Frege endlessly complains against the second while hardly mentioning the first is indicative only of the kind of misunderstanding he encountered amongst his contemporaries. (2004, 717)

Whereas Burge assigns a pragmatic role to the epistemic significance of our inferential practice for Frege, Sullivan insists that it can play no role whatsoever. His concern is that emphasizing our inferential practice results with a conception of logic—call it *sociologism*—that falls to *mutatis mutandis* variations of Frege's own criticisms of psychologism. Frege would certainly balk at allowing contingent features of our inferential practice to contaminate objective truth. Has my reading convicted Frege of the charge Burge's charitable reading sought to avoid?

To answer this question, we must first clarify Frege's objection to psychologism. Frege argues that since psychological logicians view logical laws as empirically discoverable generalizations about what most human judges hold true, their logical laws only issue in conditional prescriptions for those who wish to judge in accordance with the majority of human judges. Were human brain chemistry to undergo a radical alteration such that people began to regularly contradict themselves, for instance, a psychological logician's law of non-contradiction would have to be replaced by a law of contradiction. In contrast, Frege insists that genuine logical laws issue in unconditional prescriptions for all judges, because they unfold the content of the word 'true,' which is the constitutive goal of the practice of judgment (PW, 3).

The objectionable sociologism worrying Sullivan, then, would be to view logical laws as empirically discoverable generalizations about what most judges engaging in our linguistic practice hold true. Were our inferential practice to undergo a radical shift, 'sociological' logical

laws—and with them, objective truth—would alter. This position is certainly foreign to Frege, and is irreconcilable with his insistence that the logical laws are 'eternal boundary stones' for our thought (GGZ1, xvi).

To begin defusing Sullivan's objection, it will help to pick up his remark that Frege 'hardly mentions' the evident problems of making essential reference to our practice in an account of logic (2004, 717). This implies that Frege makes some mention of these problems—but where? Although Sullivan does not say, a related objection from Burge (1992, 642) suggests that he may be thinking of a passage where Frege criticizes Achelis' claim that

> [discovering] the norms which hold in general for thinking and acting…[requires] an empirico-critical determination of the objective principles of our psycho-physical organization which are valid at all times for the great consciousness of mankind. (quoted in PW, 146-147)

In rehearsing his argument against the psychologism evidenced by Achelis' view, Frege writes in terms that might seem broad enough to constitute a criticism of Sullivan's imagined sociologism. For he there says that were logical norms 'norms only because we seldom deviate from them,' then since what we 'normally' do can change, we must accept (absurdly, he thinks) that logical laws could change. It is not difficult to hear this as an objection both to psychological logicians who hold that we 'seldom deviate' from logical laws because of our brain chemistry, and to sociological logicians who hold that we 'seldom deviate' from logical laws because we belong to an inferential practice.

Yet this mistakes the force of Frege's objection. Achelis thought that logical laws derive their *authority* from how we actually think and judge. Sullivan assumes that to emphasize the role of our inferential practice in a conception of logic is likewise to insist that logical laws derive their authority from our actual inferential behavior. This sort of sociologism would indeed face a parallel objection to psychologism. Its proponents would owe us an account of why logical laws would not shift with changes in our inferential practice. But Frege does not think, as the

sociological logician does, that logical laws are objectively true (and derive their authority over our thought) *because* they are implicit in our inferential behavior. Rather, he thinks that logical laws are implicit in our inferential behavior (and have authority over our thought) *because* they are objectively true—that is, because they unfold the shared, objective norm that is the constitutive goal of our demonstrative inferential practice.[31] In Frege's view, if for some reason our inferential behavior systematically changed—we accepted the axiom of choice as valid, perhaps, or no longer accepted existential generalizations in the absence of witnesses—logical laws would not shift to match our new behavior. Our core logical laws are in reflective equilibrium and stand firm in their eternal foundation.

To put the point another way, Sullivan, like Burge, is wrong to think that Frege believes our practice of demonstrative inference good only insofar as it accords with a standard of objective truth that can be understood independently of our practice. Since Frege thinks that objective truth cannot be understood except through our inferential practice, the logician trying to codify the laws of truth informing that practice only has the inferential patterns we reflectively endorse to go on. Frege does not separate the capacity to engage in our inferential practice from grasping objective truth, and so, does not lapse into sociologism by purporting to ground objective truth in inferential practice. Rather, since he thinks engaging in our inferential practice and grasping objective truth to be one and the same, he sees no problem in sometimes redescribing the target of his codification of the principles implicit in our practice in terms of truth, the constitutive goal of that practice.

[31] Frege introduces his criticism of Achelis by insisting that we 'be wary of the view that it is the business of logic to investigate how we actually think and judge when we are in agreement with the laws of truth,' because in doing so we 'should have constantly to have one eye on the one thing and one eye on the other,' and eventually 'be seduced into asking questions with no clear meaning' (PW, 146). The sociological logician must face the question: might all beings engaged in our inferential practice judge this logical law true, and yet it be false? This question lacks a clear meaning since truth, as the constitutive goal of judgment, is not something judges could mistakenly attribute to basic principles of their inferential practice. In contrast, Fregean logicians *only* have their eyes on our demonstrative inferential behavior.

Sullivan recoils from Thomas Ricketts' (1986) rhetoric that our linguistic practice 'funds' Frege's conception of logic since, if participating in a practice and acknowledging its constitutive norm are one and the same, neither can be understood in terms of the other (2004, 717). But Frege's conception of logic is not 'funded' by our practice in the senses Sullivan imagines ('securing,' 'grounding,' 'supporting,' 'justifying' [717]). Rather, the grasp of objective truth, our inferential practice, and the basic Begriffsschrift laws form an instructive, interdependent circle, any of which it may be helpful to emphasize in explaining Frege's project, and to none of which we have independent access. As judges, we do not need to break into this circle; we are already inside it.

In the context of Frege's constitutive view, the inferential ability of judges manifests their certain grasp of objective truth. Our inferential practice is not merely one genus of a larger species, but the sole practice of rational judgment. Frege thus avoids sociologism, because he denies participating in our inferential practice to have a justificatory role in explaining our grasp of objective truth. Nor does he take reflecting upon our inferential practice to pragmatically, fallibly justify believing that we grasp the laws of objective truth. Frege's aim in developing a codification of the principles of inference, a codification that neither needs nor admits of justification, is simply to achieve a better understanding of the norms that govern our practice of judgment—which is to say, *the* practice of judgment.

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